Revenue Management with Reallocation

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Motivation

Many firms seek to maximize revenue by allocating heterogeneous goods to sequentially arriving consumers before a deadline:

- E.g. seasonal goods, travel and leisure industries

Dynamic pricing often appears in these settings

- Trade-off: Allocating today may forego more valuable future allocation

$\Longrightarrow$ Allocation may be ex-post inefficient!

Motivation

Reallocation can reduce ex-post inefficiencies.

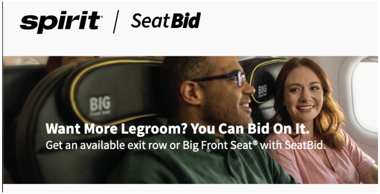
Vertically-differentiated products allow for reallocation via upgrades:

- Trade-off: Collect upgrade revenue and free up basic good, but forego opportunity to sell premium good later at full price

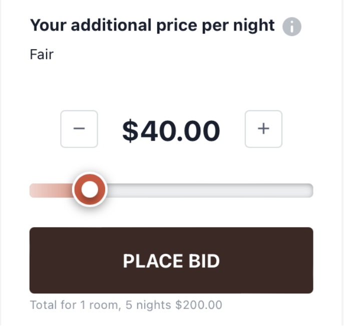
Consumers can respond strategically to upgrade mechanisms and undermine the screening intention of prices.

Upgrade Auctions

Amtrak (Rail)



Spirit Airlines



Lucerne Hotel (NYC)

Research Question and Approach

Question

What are the profit and welfare implications of using upgrade mechanisms with dynamic pricing?

Approach

- Collect and analyze novel proprietary data from an airline using dynamic pricing and upgrade auctions.
- Estimate an equilibrium model of an airline allocating seats to strategic consumers.
- Use model estimates for counterfactual calculations to examine interaction of dynamic pricing and auctions.

Literature

Dynamic and Discriminatory Pricing:

- Sweeting [2010], Williams [2022], Aryal et al. [2023], Dubé and Misra [2023]

Auctions:

- Vulcano et al. [2002], Roberts and Sweeting [2012], Gentry and Li [2014]

Revenue Management and Mechanism Design:

- Gershkov and Moldovanu [2009], Board and Skrzypacz [2016], Cui et al. [2018, 2019], Dilmé and Li [2018]

Data

Three months of data from NA airline with multiple ways to upgrade:

1. Revenue Management Data:
   - Aircraft cabin capacities and flight information
   - Daily ticket sales and transaction prices
2. Search Data:
   - Web traffic data from the airline's website
   - All flight searches and purchases from the airline's website, including redirected traffic e.g. Google Flights, Kayak
3. Upgrade Data:
   - Bids placed for upgrades, outcomes of bids, and other auction info
   - Upgrades purchased at check-in including price

Details of Upgrade Mechanisms

Upgrade auction is a first-price auction:

- Slider used to submit bid from a discrete set of values
- Upgrade decisions are made at a fixed day before departure
- Pricing polices unchanged after auction introduction

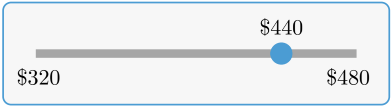
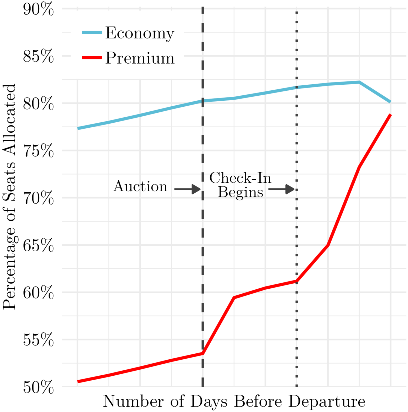
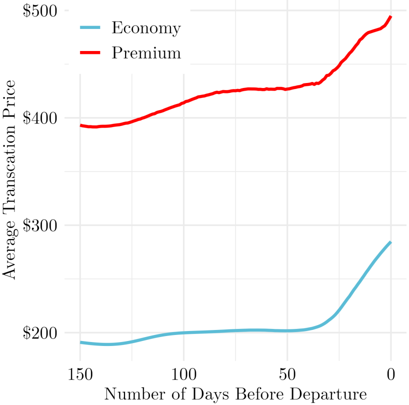


Figure 2: Example Slider

Remaining premium seats sold for a market-specific fixed fee at check-in.

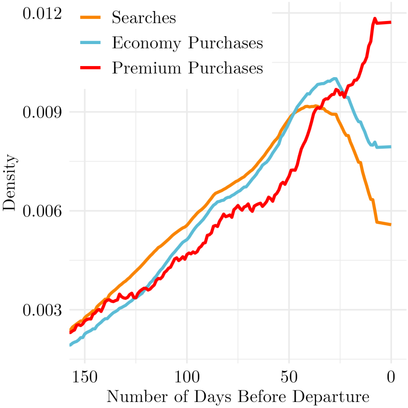
Descriptives: Flight Inventory and Fares

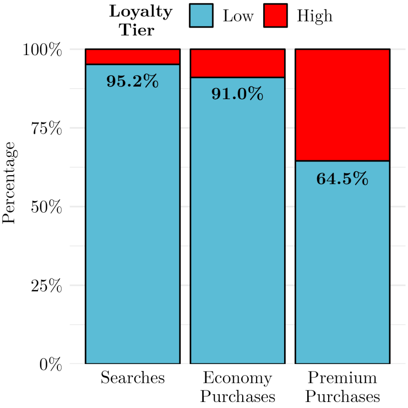




- Large share of premium seats are allocated using upgrade mechanisms
- Substantial temporal variation and class differences in fares

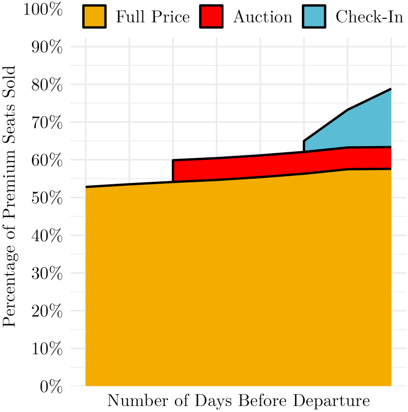
Descriptives: Arrival Patterns from Search Data

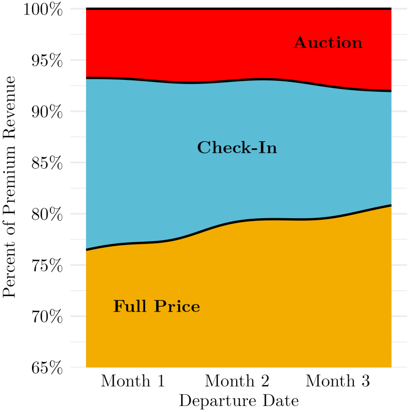




- Economy purchases generally follow pattern of searches
- Overall ratio of 14 searches to 1 purchase
- High-tier customers disproportionately purchase premium seats later

Descriptives: Upgrade Allocations and Revenue





- Upgrades account for ≈ 25% of premium ticket sales and revenue.

Model: Airline - Overview

Monopoly airline endowed with seats $\boldsymbol{k}_1 = (k_1^f, k_1^e)$ in two vertically-differentiated cabins $\{f, e\}$ maximizes profits by:

- Selling tickets in each period $t \in \{1, ..., T\}$ setting prices $\boldsymbol{p}_t = (p_t^f, p^e)$
- Allowing economy consumers to bid $b \in \{b^1, ..., b^J\}$ for upgrades
- Running auction in fixed period $\tilde{t}$ (before setting prices)
- Randomly offering remaining premium seats for price r in period $T+1$

$\implies$ Solution to dynamic programming problem yields optimal pricing and bid-acceptance policies.

Model: Airline's Pricing Problem

Revenue: $R(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k}) = p^f Q^f(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k}) + p^e Q^e(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k})$

- Demand realizations $\boldsymbol{y} = (t_i, \nu_i, \xi_i)_{i=1}^{N_t}$ with N_t being the number of consumer arrivals

Firm's dynamic program:

$$V_t(\boldsymbol{k}) \; = \; \max_{\boldsymbol{p} \in \mathbb{R}_+^2} \overbrace{\mathbb{E}_t[R(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k})]}^{\text{Expected Revenue}} + \overbrace{\delta \int_{\boldsymbol{k}' \in \mathcal{K}} V_{t+1}(\boldsymbol{k}') dH_t(\boldsymbol{k}' | \boldsymbol{k}, \boldsymbol{p})}^{\text{Continuation Value}}$$

- Revenue expectation over $\boldsymbol{y}$ for a given $\boldsymbol{k}$ and $\boldsymbol{p}$
- Solution via backwards induction yields policy function $\boldsymbol{p}_t(\boldsymbol{k})$

Model: Airline's Upgrade Decision

Denote the count of each submitted bid values at t as $\boldsymbol{\mathscr{b}}_t = (\mathscr{b}_t^1, ..., \mathscr{b}_t^J)$

If airline upgrades n passengers, state resets to $\boldsymbol{k}_{\tilde{t}} + n\boldsymbol{i^u}$

- $\boldsymbol{i^u}$ is the upgrade vector $\boldsymbol{i^u} = (-1, 1)$

1. Marginal revenue of n^{th} upgrade: n^{th} highest bid
2. Marginal cost of n^{th} upgrade: $\Delta V_{\tilde{t}}(n, \boldsymbol{k}) = V_{\tilde{t}}(\boldsymbol{k} + n\boldsymbol{i^u}) - V_{\tilde{t}}(\boldsymbol{k} + (n-1)\boldsymbol{i^u})$

If $\Delta V_{\tilde{t}}(n, \boldsymbol{k})$ is increasing in n, accept bids until the $n+1$ largest bid is smaller than $\Delta V_{\tilde{t}}(n+1, \boldsymbol{k})$.

Model: Consumers - Overview

$N_t \sim \text{Poisson}(\lambda_t)$ short-lived consumers arrive in period t and maximize expected utility:

- Choose from set $\{f, e, o\}$: premium, economy, or not traveling
- Purchase sequentially based on idiosyncratic arrival time $t_i \in [t, t+1)$

Choices depend on:

- Private valuations (ν_i, ξ_i) for travel and quality, respectively
- Beliefs of upgrade $\boldsymbol{\varrho}_t(\boldsymbol{k}) = (\varrho_t^1(\boldsymbol{k}), ..., \varrho_t^J(\boldsymbol{k}))$ with bids $b \in \{b^1, ..., b^J\}$
- Belief of check-in upgrade $\varphi_t(\boldsymbol{k})$ with fee r

$\Longrightarrow$ Solution is a Bayesian Nash Equilibrium where beliefs are consistent with airline's bid-acceptance decisions.

Model: Consumer Choice

Linear utility for choice $m \in \{f, e, o\}$: $u_{it}^m \equiv \nu_i \xi_i^m - p$

- Normalize $\xi_i^e = 1$, $\xi_i^o = 0 \implies \xi_i^f = \xi_i$
- $(\nu_i, \xi_i) \sim F_{\nu_t} \times F_{\xi_t}$ with $\nu_i \geq 0$, $\xi_i \geq 1$

Optimal bid b_{it}^* with belief $\varrho_{it}^*(\boldsymbol{k})$ maximizes expected utility:

$$\mathcal{U}_{it}^e = \underbrace{\nu_i + p_t^e}_{\text{Certain utility}} + \underbrace{\varrho_{it}^*(\boldsymbol{k})\big(\nu_i(\xi_i - 1) - b_{it}^*\big)}_{\text{Auction utility}} + \overbrace{\Big(1 - \varrho_{it}^*(\boldsymbol{k})\Big)\varphi_t(\boldsymbol{k})\underbrace{\max\{0, \nu_i(\xi_i - 1) - r\}}_{\text{Willing to pay check-in}}}^{\text{Check-in utility}}$$

Consumers choose from $\{f, e, o\}$ by comparing u_{it}^f, $\mathcal{U}_{it}^e$, and $u_{it}^o \equiv 0$.

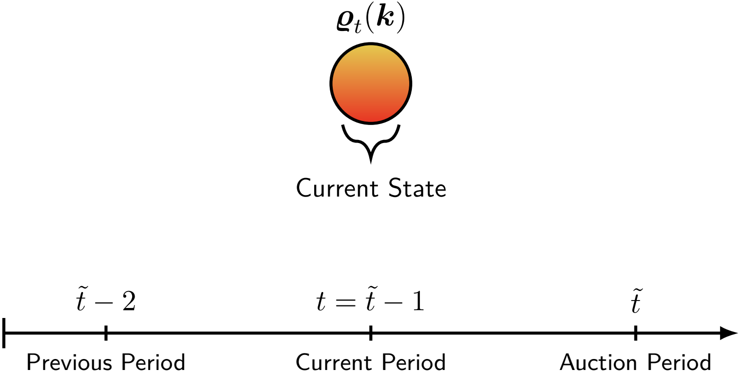
Model: Calculating Equilibrium Beliefs

Calculating equilibrium beliefs is challenging given non-stationary environment, dimensionality of state space, and selection into auction

Develop an iterative forward-simulation procedure to solve numerically:

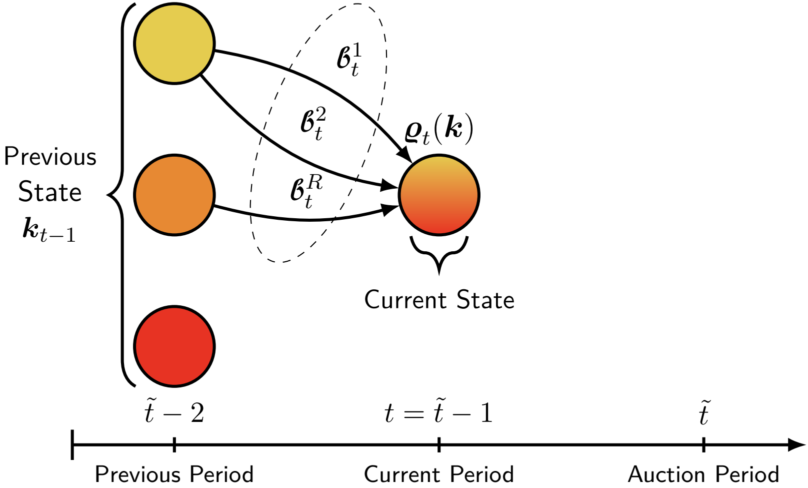
1. Simulate sequence of decisions given initial beliefs
2. Calculate upgrade probabilities given airline's bid-acceptance policy
3. Update initial beliefs to equal win probabilities
4. Iterate 1, 2, and 3 until convergence to a fixed point

Model: Calculating State-Specific Equilibrium Beliefs



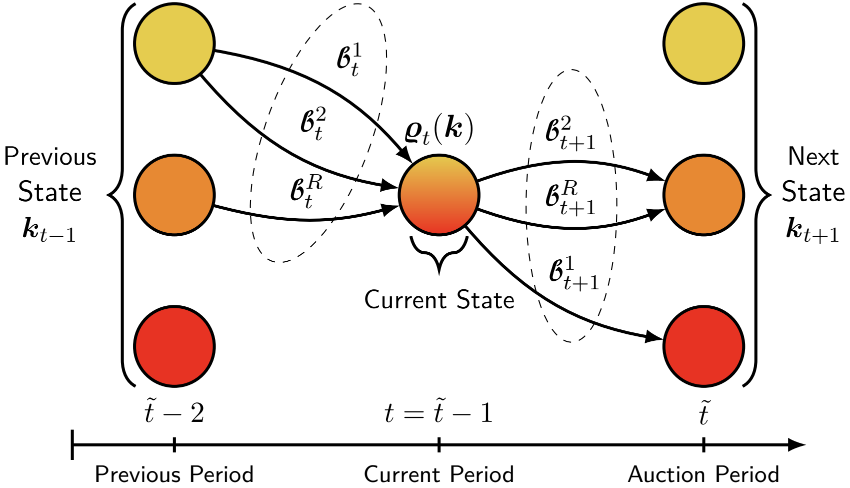
– Consumers arriving at $t=\tilde{t}-1$ in state $\boldsymbol{k}$ form upgrade beliefs $\boldsymbol{\varrho}_t(\boldsymbol{k})$

Model: Calculating State-Specific Equilibrium Beliefs



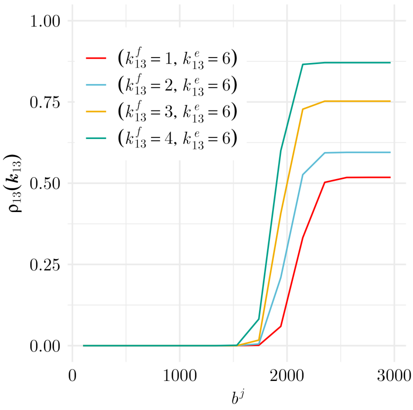
- Potential paths through the node have unobserved collection of bids $\boldsymbol{b}_t$

Model: Calculating State-Specific Equilibrium Beliefs

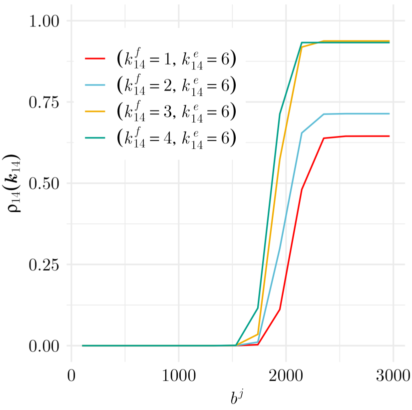


- $\boldsymbol{\mathscr{b}}_t$ and $\boldsymbol{k}_t$ evolve given choices and determine upgrade decisions at $\tilde{t}$

Model: Example of Equilibrium Beliefs



Beliefs at $t = 13$



Beliefs at $t = 14$

- State-specific beliefs for auction at $\tilde{t} = 15$ exhibit expected properties

Next Steps: Estimation and Counterfactuals

Currently estimating the model using flexible method of moments approach

– Fox et al. [2016], Ackerberg [2009], Aryal et al. [2023]

Perform counterfactuals using estimates:

1. Optimize features of auction like timing and reserve price
2. Update pricing policies to account for upgrade mechanisms
3. Allow pricing policies to depend on collected bids
4. Allow losing bids to influence price or probability of check-in upgrade

THANK YOU!